

Modulational Instability of Polarized Electromagnetic Waves in Cylindrical Fibers

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Abstract

The polarized electromagnetic wave in cylindrical fibers is described by a system of four component vector nonlinear Schrödinger equations. We have investigated analytically and numerically the modulational instability of this system. Several types of modulational instability are shown to exist depending on the sign of the dispersion coefficients. The results indicate that the group-velocity mismatch and the four wave mixing terms influence considerably the shape, position and especially the number of the spectral regions of the gain spectra of the modulation instability. Numerical simulations of the four component vector nonlinear Schrödinger equation show that the phenomenon of modulational instability gives rise to periodic pulse arrays of waves train. The presence of the four wave mixing terms influences considerably the evolution of the periodic pulse.

Keywords

Modulation Instability; Cross-phase Modulation; Walk-off Effect; Four Wave Mixing

Introduction

Vector solitons in multiple waveguides have drawn tremendous attention recently. While solitons can propagate over a long distance from a balance of dispersion and nonlinearity, coupling in two or more waveguides [1] permits additional flexibility which generates a rich variety of phenomena. The study of the dynamics of nonlinear waves is central to understand the behaviour of a wide range of physical systems, taking reference to [2], for example. In recent years, an important theme of much of this work has

been the study of modulational instabilities (MI) of plane waves. MI occurs as a result of the interaction between the linear dispersive effect and the nonlinear effect, which is characterized by instability experienced by a continuous wave (CW) when it propagates inside a dispersive nonlinear system together with a weak noise. The perturbed wave experiences exponential growth and this leads to the breakup of the temporal profile of a wave into a train of ultrashort pulses [3, 4]. Since this disintegration typically occurs in the same parameter region where bright solitons are observed, MI is considered to some extent, a precursor to soliton formation in nonlinear systems [3]. The MI gain has been extensively studied for the scalar nonlinear Schrödinger type equations (NLSEs) [4]. The experimental evidence was then given that MI can be observed in a non-birefringent fiber with anomalous group velocity dispersion (GVD) [5]. This result has been confirmed subsequently by other observations of MI in the anomalous dispersion regime [6]-[7]. Vector MI can occur in isotropic fibers such that the gain spectrum depends on the polarization state of the input CW beam. It has been known, however, since the early studies by Bespalov and Talanov [8], and Berkhoer and Zakharov [9], that an incoherent (i.e., purely intensity dependent) coupling between two NLSE leads to extension of the instability domain in the normal dispersion regime. Unfortunately, it is very difficult to manufacture birefringence-free fibers. As an alternative, MI was observed in a bimodal fiber in which the input beam

excited two fiber modes with nearly equal power levels, and the two modes had the same group velocity [10]. However in a multi-mode fiber the velocity difference (walk-off) has to be taken into account, because different modes have substantially different group velocities. Previous studies have shown that the effect of walk-off among optical pulses can considerably modify the gain spectra of cross-phase modulation (XPM) induced MI [11]. So far, a different situation that is of considerable interest in nonlinear optics is the coherent coupling between two polarizations in a weakly anisotropic media. Coherent coupling can arise in the polarization of light beams, as the relative phase factors of the interacting electric fields there play a crucial role in the form of four-wave mixing (FWM) process. FWM in optical fibers can be both harmful and beneficial depending on the type of application. It can induce crosstalk in wavelength division multiplexing communication systems and limit the performance of such systems. The FWM-induced crosstalk can be minimized in practice through dispersion management, a technique in which the dispersion of each fiber section is made large enough that the FWM process is not phase matched through-out the link length [12]. At the same time, FWM is useful for variety of applications [13]. FWM can be useful for applications such as optical sampling, channel demultiplexing, pulse generation, and high-speed optical switching [14]. It can also be used for reducing quantum noise through squeezing, and for generating photon pairs that are quantum-correlated. The resulting model is a system of coherently coupled NLSE (CNLSE). In the light of the above mentioned facts, it is worthwhile to study the phenomenon of MI in a fiber system with the FWM nonlinearity. Recently, MI in a system of coherently CNLSE has been investigated and it was found that the nonlinearity arising from phase coherence enlarges the range of XPM-induced MI [15]. Modulation instabilities arising from systems of four CNLSE have also been studied earlier in the literature [16]. It was found here that MI can appear even in the regime where the XPM coefficient is less than that of the self-phase modulation (SPM). The group-velocity mismatch (GVM) and the mixed interaction between different modes (FWM) have been neglected. Therefore, generally speaking, both of the walk-off effects and FWM terms should be taken into consideration in the study of XPM induced-MI in the four CNLSE. Our motivation in the present work is to study the MI of the four CNLSE, derived by Leble and Reichel [17].

The remainder of the article is arranged as follows. In section 2, a system of four CNLSE is presented. In section 3, we study the linear stability analysis of this model equation. The results of MI gain are investigated in section 4. Typical outcomes of the nonlinear development of the MI are reported in section 5 and section 6 concludes the paper.

The model

Wave propagation for pulses of polarized electromagnetic waves in cylindrical fibers is governed by a CNLSE [17]

$$\frac{\partial X_{01}^{\pm}}{\partial z} = \alpha_{01}^{\pm} \frac{\partial X_{01}^{\pm}}{\partial t} + i\beta_{01}^{\pm} \frac{\partial^2 X_{01}^{\pm}}{\partial t^2} - i \left[O_1 |X_{01}^{\pm}|^2 + O_2 |X_{01}^{\mp}|^2 + O_3 |X_{11}^{\pm}|^2 + O_4 |X_{11}^{\mp}|^2 + O_5 X_{11}^{\mp} X_{11}^{\pm*} + O_6 X_{11}^{\pm} X_{11}^{\mp*} \right] X_{01}^{\pm},$$

$$\frac{\partial X_{11}^{\pm}}{\partial z} = \alpha_{11}^{\pm} \frac{\partial X_{11}^{\pm}}{\partial t} + i\beta_{11}^{\pm} \frac{\partial^2 X_{11}^{\pm}}{\partial t^2} - i \left[Q_1 |X_{11}^{\pm}|^2 + Q_2 |X_{11}^{\mp}|^2 + Q_3 |X_{01}^{\pm}|^2 + Q_4 |X_{01}^{\mp}|^2 + Q_5 X_{01}^{\mp} X_{01}^{\pm*} + Q_6 X_{01}^{\pm} X_{01}^{\mp*} \right] X_{11}^{\pm}. \quad (1)$$

Here, $X_{01}^{\pm}$ and $X_{11}^{\pm}$ are the pulses envelope in each polarization mode (two modes and each mode has two polarizations). $\alpha_{j1}^{\pm}$ and $\beta_{j1}^{\pm}$ ($j=0, 1$) denote the GVM of pulses and the GVD, respectively. O_1 and Q_1 are SPM coefficients. O_2, O_3, O_4, Q_2, Q_3 , and Q_4 are the XPM coefficients. The nonlinear coefficients O_5, O_6, Q_5 and Q_6 describe mixed interaction between different modes and different polarization (FWM). In the limit of high birefringence, the FWM mixing terms in Eq. (1) can be neglected as they can be averaged to zero. In contrast, these coherent phase nonlinearities will be retained in the present work. Although solitons have been studied intensively [18-21], the focus here will be on the MI of plane waves of Eq. (1). Many existing works have assumed the low birefringence limit. The first objective here is to demonstrate that restoring this physics changes considerably the MI sidebands. Secondly, inclusion of phase dependent nonlinearity will be shown to extend the XPM-induced MI of the CNLSE.

The linear stability analysis

In order to investigate the evolution of weak perturbations along the propagation distance, we carry out a linear-stability analysis. Firstly, we look for CW solution of the system described by Eq. (1) given by:

$$X_{j1}^{\pm} = \sqrt{P_{j1}^{\pm}} \exp[i(\phi_j^{\pm} - i\phi_j z)], \quad j=0, 1 \quad (2)$$

where $P_{j1}^{\pm}$ is the power of the plane wave and the real phase $\phi_j^{\pm}$ are given by:

$$\phi_0 = O_1 |P_{01}^{\pm}|^2 + O_2 |P_{01}^{\mp}|^2 + O_3 |P_{11}^{\pm}|^2 + O_4 |P_{11}^{\mp}|^2 + O_5 \sqrt{P_{11}^{\mp} P_{11}^{\pm}} + O_6 \sqrt{P_{11}^{\pm} P_{11}^{\mp}}$$

$$\phi_1 = Q_1 |P_{11}^{\pm}|^2 + Q_2 |P_{11}^{\mp}|^2 + Q_3 |P_{01}^{\pm}|^2 + Q_4 |P_{01}^{\mp}|^2 + Q_5 \sqrt{P_{01}^{\mp} P_{01}^{\pm}} + Q_6 \sqrt{P_{01}^{\pm} P_{01}^{\mp}} \quad (3)$$

In order to examine the linear-stability of the CW states, we introduce the perturbed fields of the form

$$X_{j1}^{\pm} = (\sqrt{P_{j1}^{\pm}} + a_{j1}^{\pm}) \exp(-i\phi_j z), \quad j=0, 1 \quad (4)$$

Where $a_{j1}^{\pm} \ll \sqrt{P_{j1}^{\pm}}$. By substituting Eq. (1) and retaining linear terms of $a_{j1}^{\pm}$, we obtain four linearized equations:

$$\begin{aligned} \frac{\partial a_{01}^{\pm}}{\partial z} = & a_{01}^{\pm} \frac{\partial a_{01}^{\pm}}{\partial t} + i\beta_{01}^{\pm} \frac{\partial^2 a_{01}^{\pm}}{\partial t^2} - i[O_1 P_{01}^{\pm} (a_{01}^{\pm} + a_{01}^{\pm*}) + \\ & O_2 \sqrt{P_{01}^{\pm} P_{01}^{\mp}} (a_{01}^{\mp} + a_{01}^{\mp*}) + O_3 \sqrt{P_{11}^{\pm} P_{01}^{\mp}} (a_{11}^{\mp} + a_{11}^{\mp*}) + \\ & O_4 \sqrt{P_{11}^{\mp} P_{01}^{\mp}} (a_{11}^{\mp} + a_{11}^{\mp*}) + O_5 \sqrt{P_{01}^{\pm} P_{11}^{\mp}} a_{11}^{\mp} + O_5 \sqrt{P_{01}^{\mp} P_{11}^{\mp}} a_{11}^{\pm*} + \\ & O_6 \sqrt{P_{01}^{\pm} P_{11}^{\mp}} a_{11}^{\mp} + O_6 \sqrt{P_{01}^{\mp} P_{11}^{\mp}} a_{11}^{\mp*}], \end{aligned}$$

$$\begin{aligned} \frac{\partial a_{11}^{\pm}}{\partial z} = & -a_{11}^{\pm} \frac{\partial a_{11}^{\pm}}{\partial t} + i\beta_{11}^{\pm} \frac{\partial^2 a_{11}^{\pm}}{\partial t^2} - i[Q_1 P_{11}^{\pm} (a_{11}^{\pm} + a_{11}^{\pm*}) + \\ & Q_2 \sqrt{P_{11}^{\pm} P_{11}^{\mp}} (a_{11}^{\mp} + a_{11}^{\mp*}) + Q_3 \sqrt{P_{11}^{\mp} P_{01}^{\mp}} (a_{01}^{\mp} + a_{01}^{\mp*}) + \\ & Q_4 \sqrt{P_{01}^{\mp} P_{11}^{\mp}} (a_{01}^{\mp} + a_{01}^{\mp*}) + Q_5 \sqrt{P_{01}^{\pm} P_{11}^{\mp}} a_{01}^{\mp} + Q_5 \sqrt{P_{11}^{\mp} P_{01}^{\mp}} a_{01}^{\pm*} + \\ & Q_6 \sqrt{P_{11}^{\mp} P_{11}^{\mp}} a_{01}^{\pm} + Q_6 \sqrt{P_{01}^{\pm} P_{11}^{\mp}} a_{01}^{\mp*}], \end{aligned} \quad (5)$$

where * denote complex conjugate. We assume general solutions of the form

$$a_{j1}^{\pm} = u_{j1}^{\pm} \exp(i\Omega t) + v_{j1}^{\pm} \exp(-i\Omega t), \quad j=0, 1 \quad (6)$$

Inserting Eq. (5), one obtains a set of eight linear ordinary differential equations

$$\frac{d[Y]}{dz} = i[M][Y] \quad (7)$$

where [M] is an 8×8 matrix with

$[Y] = [u_{01}^+, v_{01}^{+*}, u_{01}^-, v_{01}^{-*}, u_{11}^+, v_{11}^{+*}, u_{11}^-, v_{11}^{-*}]^T$. The elements of [M] are given in appendix. The above 8×8 matrix is referred to as stability matrix which is used to study the stability of the system under consideration. MI occurs only when at least one of the eigenvalues of the stability 8×8 matrix possesses a nonzero and negative

imaginary part, which results in an exponential growth of the amplitude with the perturbation. By requiring the determinant of the associated matrix in Eq. (7) to vanish, we can find the solvability condition for these equations, which amounts to a polynomial of degree eight. The MI phenomenon is measured by a power gain given by $G = 2|Im(K)|$ where K is the eigenvalue of MI that possesses the largest imaginary part [22].

The Modulational Instability Gain

The MI gain will be calculated and discussed for different dispersion regions in the following. To simplify the discussion, we set the incident optical powers of the four optical waves as $P_{j1}^{\pm} = P$. We set also the GVM as $\alpha_{01}^{\pm} = -\alpha_{11}^{\pm} = \delta$. We have also chosen the following physical parameters [23]: $\beta_{j1}^{\pm} = 0.06 \text{ ps}^2/\text{m}$ ($j=0, 1$), $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5 \text{ W}$. The parameter $\delta = 2 - 5 \text{ ps/m}$ for a wavelength difference 5 - 10nm. Several qualitatively different situations emerge depending on the signs of dispersion coefficients.

$$\text{Case 1: } \beta_{01}^+ < 0, \beta_{01}^- < 0, \beta_{11}^+ < 0, \beta_{11}^- < 0$$

In this case, all the four beams propagate in the anomalous GVD of the fiber. This case is of particular importance since each beam can develop MI even in the absence of the XPM coupling. The gain spectra of MI for different parameters δ have been shown in FIG. 1. We have obtained in this case in FIG. 1 (a1) only one sideband.

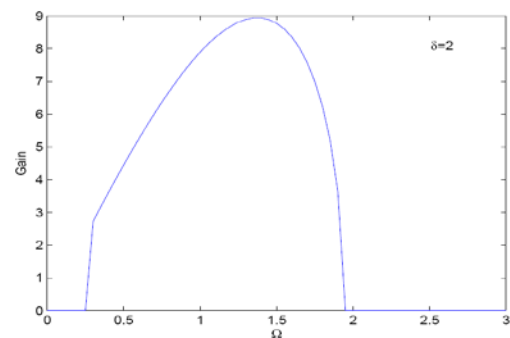


FIG. 1(A1) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FOUR BEAMS EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06 \text{ Ps}^2/\text{M}$, $\beta_{11}^{\pm} = -0.06 \text{ Ps}^2/\text{M}$, $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5 \text{ W}$

As δ increases, the number of the spectral peak increases also as seen in FIG. 1 (a2),

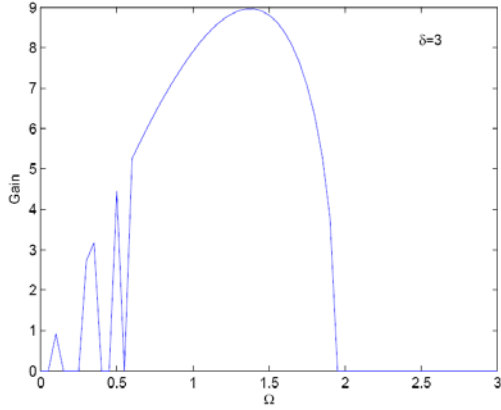


FIG. 1(a2) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FOUR BEAMS EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$, $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5\text{ W}$

and on further increasing δ , we observe in FIG. 1 (a3) that the additional sidebands are suppressed.

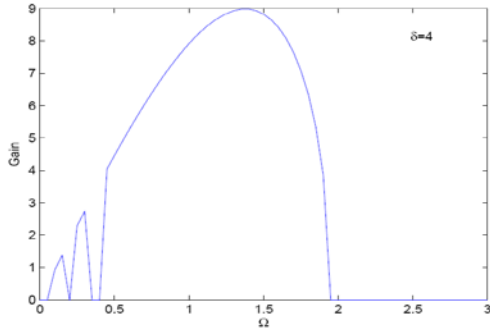


FIG. 1(a3) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FOUR BEAMS EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$, $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5\text{ W}$

For the large values of δ , we observe in FIG. 1 (a4) only one sideband.

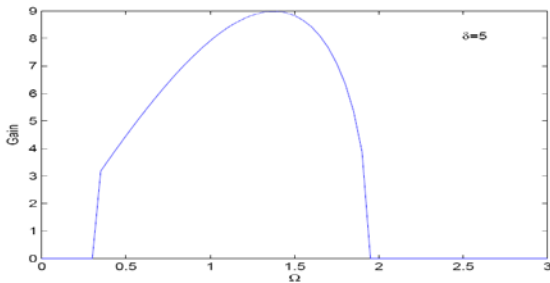


FIG. 1(a4) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FOUR BEAMS EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$, $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5\text{ W}$

By comparison in FIG. 1 (b) where we have taken into account the FWM nonlinear coefficients, the gain spectrum takes on new characteristics and the number of the spectral peak are quite different from those of FIG. 1 (a).

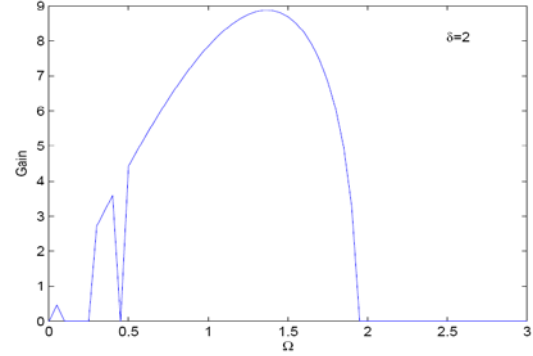


FIG. 1(b1) THE MI GAIN SPECTRA IN THE CASE WHERE THE FWM NONLINEARITIES ARE TAKEN INTO ACCOUNT. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$, $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5\text{ W}$

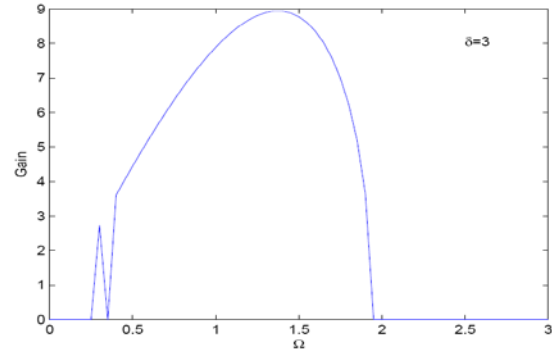


FIG. 1(b2) THE MI GAIN SPECTRA IN THE CASE WHERE THE FWM NONLINEARITIES ARE TAKEN INTO ACCOUNT. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$, $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5\text{ W}$

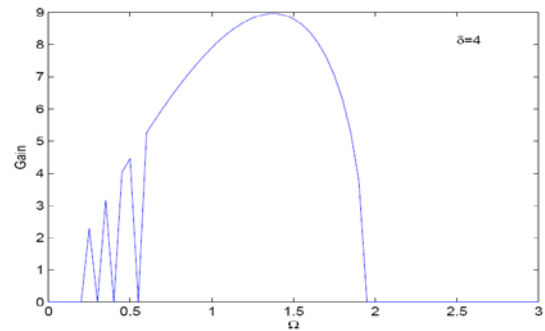


FIG. 1(b3) THE MI GAIN SPECTRA IN THE CASE WHERE THE FWM NONLINEARITIES ARE TAKEN INTO ACCOUNT. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$, $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5\text{ W}$

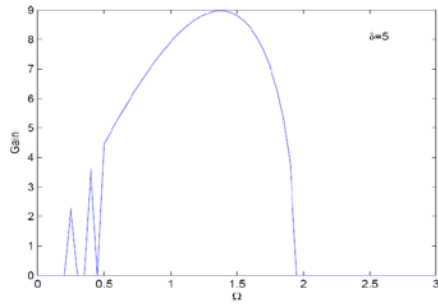


FIG. 1(b4) THE MIGAIN SPECTRA IN THE CASE WHERE THE FWM NONLINEARITIES ARE TAKEN INTO ACCOUNT. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$, $O_1 = Q_1 = 1$, $O_2 = Q_2 = 2$, $O_3 = O_4 = Q_3 = Q_4 = 2/3$, $O_5 = O_6 = Q_5 = Q_6 = 1/3$, $P = 1.5 \text{ W}$

Case 2: $\beta_{01}^{+} < 0, \beta_{01}^{-} < 0, \beta_{11}^{+} > 0, \beta_{11}^{-} > 0$

In this case, the first mode experiences anomalous GVD, while the second mode propagates in the normal GVD region. When $\delta = 2$, we observe in FIG. 2 (a1) only one spectral peak,

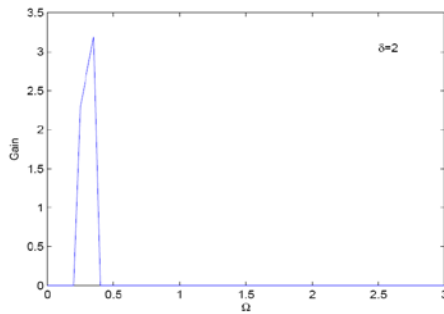


FIG. 2(a1) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FIRSTMODE EXPERIENCES ANOMALOUS GVD, WHILE THE SECOND MODE PROPAGATES IN THE NORMAL DISPERSION REGIME. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = 0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

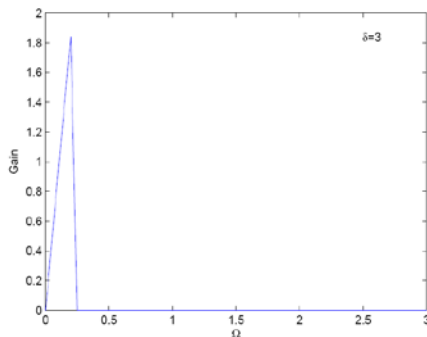


FIG. 2(a2) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FIRSTMODE EXPERIENCES ANOMALOUS GVD, WHILE THE SECOND MODE PROPAGATES IN THE NORMAL DISPERSION REGIME. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = 0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

but with the increase of the parameter δ , we observe in FIG. 2 (a2) that the position of the spectral peak varies, and in FIG. 2 (a3) that the number of the spectral region increases.

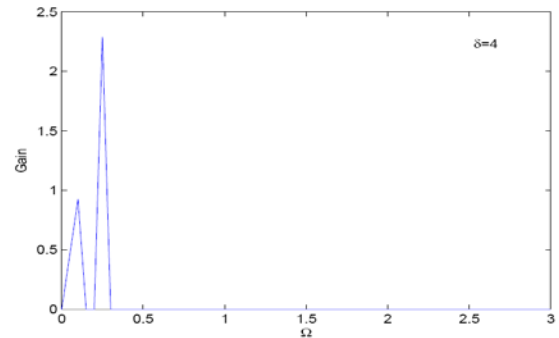


FIG. 2(a3) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FIRSTMODE EXPERIENCES ANOMALOUS GVD, WHILE THE SECOND MODE PROPAGATES IN THE NORMAL DISPERSION REGIME. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = 0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

For $\delta \geq 5$, we have seen that the spectral peaks are totally suppressed. When we take into account the FWM nonlinear coefficients, we observe in FIG. 2 (b1)-FIG. 2(b3), that these coefficients alter the number and the position of the spectral peaks.

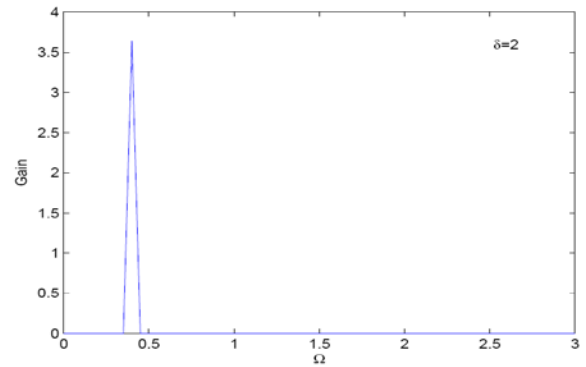


FIG. 2(b1) THE MI GAIN SPECTRA IN THE PRESENCE OF THE FWM NONLINEARITIES. THE FIRSTMODE EXPERIENCES ANOMALOUS GVD, WHILE THE SECOND MODE PROPAGATES IN THE NORMAL DISPERSION REGIME. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = 0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

Case 3: $\beta_{01}^{+} > 0, \beta_{01}^{-} > 0, \beta_{11}^{+} < 0, \beta_{11}^{-} < 0$

In this case, the first mode experiences normal GVD, while the second mode propagates in the anomalous GVD region. The gain spectra of MI for different parameters δ have been shown in FIG. (3). When $\delta = 2$, the gain spectrum in FIG. 3 (a1) contains four spectral regions.

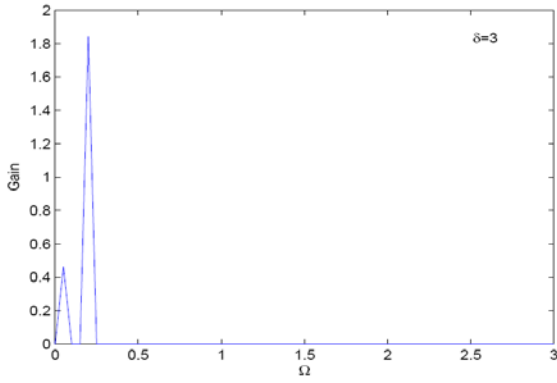


FIG. 2(b2) THE MI GAIN SPECTRA IN THE PRESENCE OF THE FWM NONLINEARITIES. THE FIRST MODE EXPERIENCES ANOMALOUS GVD, WHILE THE SECOND MODE PROPAGATES IN THE NORMAL DISPERSION REGIME. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = 0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

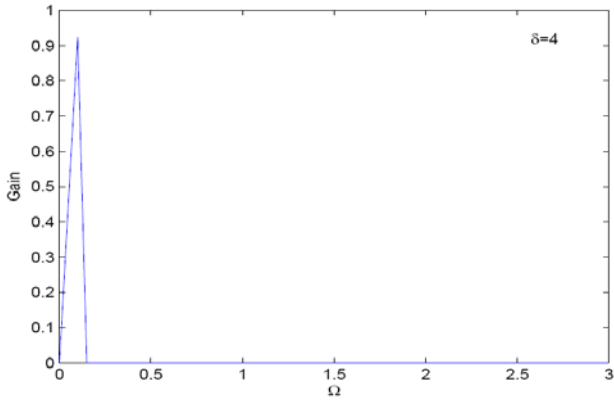


FIG. 2(b3) THE MI GAIN SPECTRA IN THE PRESENCE OF THE FWM NONLINEARITIES. THE FIRST MODE EXPERIENCES ANOMALOUS GVD, WHILE THE SECOND MODE PROPAGATES IN THE NORMAL DISPERSION REGIME. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = 0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1.

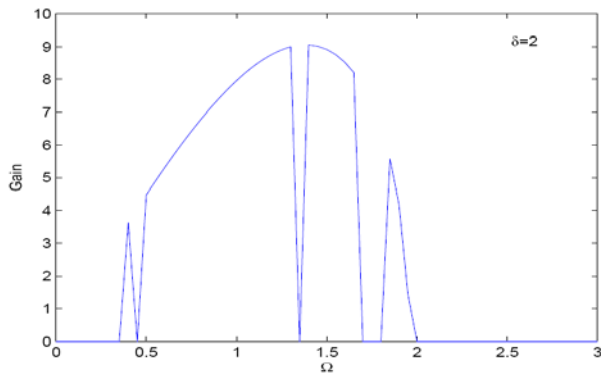


FIG. 3(a1) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FIRST MODE PROPAGATES IN THE NORMAL DISPERSION REGIME, WHILE THE SECOND MODE EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

With the increase of the parameter δ , it can be realized that the number of the spectral peak decreases and we observe only two sidebands. For the large values of δ , the bandwidth of the second sideband increases, while the peak gain of the first sideband decreases (FIG. 3(a2)- (a4)).

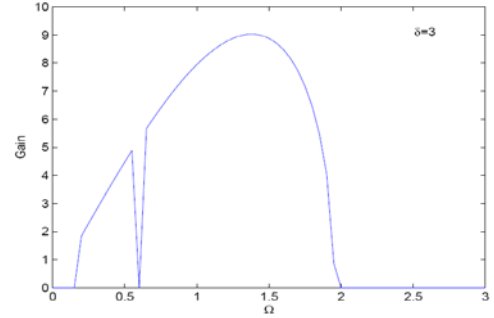


FIG. 3(a2) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FIRST MODE PROPAGATES IN THE NORMAL DISPERSION REGIME, WHILE THE SECOND MODE EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1.

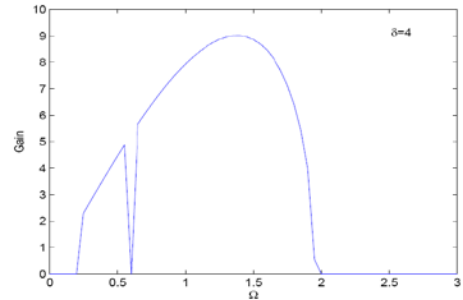


FIG. 3(a3) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FIRST MODE PROPAGATES IN THE NORMAL DISPERSION REGIME, WHILE THE SECOND MODE EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1.

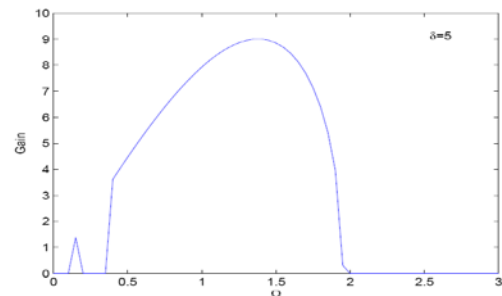


FIG. 3(a4) THE MI GAIN SPECTRA WITHOUT THE FWM NONLINEARITIES. THE FIRST MODE PROPAGATES IN THE NORMAL DISPERSION REGIME, WHILE THE SECOND MODE EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1.

Furthermore, Fig. 3 (b1)-Fig. 3 (b4) show also that with the presence of the FWM coefficients, the spectral peaks are different with those obtained in Fig. 3 (a).

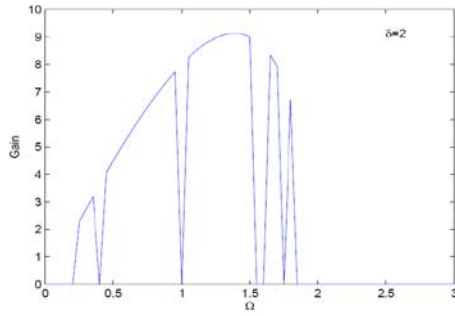


FIG. 3(b1) THE MIGAIN SPECTRA IN THE CASE WHERE THE FWM NONLINEARITIES ARE TAKEN INTO ACCOUNT. THE FIRST MODE PROPAGATES IN THE NORMAL DISPERSION REGIME, WHILE THE SECOND MODE EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

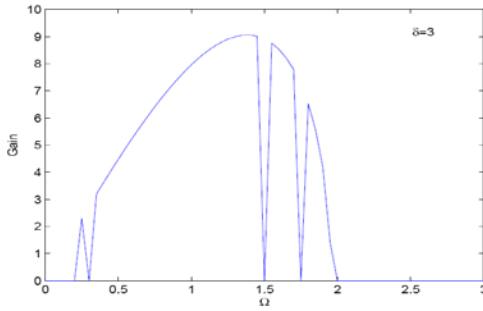


FIG. 3(b2) THE MIGAIN SPECTRA IN THE CASE WHERE THE FWM NONLINEARITIES ARE TAKEN INTO ACCOUNT. THE FIRST MODE PROPAGATES IN THE NORMAL DISPERSION REGIME, WHILE THE SECOND MODE EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

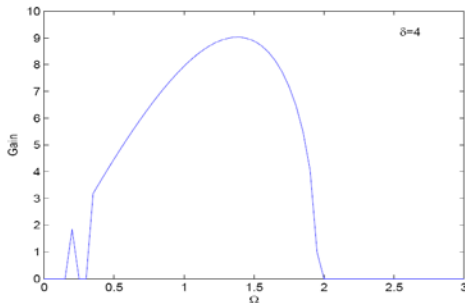


FIG. 3(b3) THE MIGAIN SPECTRA IN THE CASE WHERE THE FWM NONLINEARITIES ARE TAKEN INTO ACCOUNT. THE FIRST MODE PROPAGATES IN THE NORMAL DISPERSION REGIME, WHILE THE SECOND MODE EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

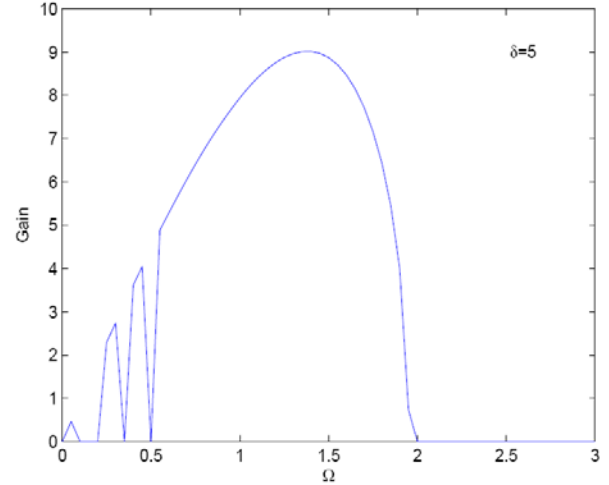


FIG. 3(b4) THE MIGAIN SPECTRA IN THE CASE WHERE THE FWM NONLINEARITIES ARE TAKEN INTO ACCOUNT. THE FIRST MODE PROPAGATES IN THE NORMAL DISPERSION REGIME, WHILE THE SECOND MODE EXPERIENCES ANOMALOUS GVD. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

Numerical Simulations in Different GVD Domains

Direct numerical simulations were run in order to understand the dynamics of CW states under the MI in the nonlinear regime. We have solved Eq. (1) using a split-step Fourier method applying the fast Fourier transforms [24]. The numerical simulations have been carried out using various step sizes down to 0.0001 throughout the z direction, with up to 1024 points along the t direction. The CW state was initially perturbed as:

$$X_{01}^{\pm} = \sqrt{P_0}(1 \pm 0.005)\cos(\Omega t)$$

$$X_{11}^{\pm} = \sqrt{P_0}(1 \pm 0.005)\cos(\Omega t) \quad (8)$$

where $P_0 = 1.5$ is the power of the CW and $\Omega = 0.2$ is the angular frequency of the weak sinusoidal modulation imposed on the CW. Examples of different numerical results are shown below.

$$\text{Case 1: } \beta_{01}^{+} < 0, \beta_{01}^{-} < 0, \beta_{11}^{+} > 0, \beta_{11}^{-} > 0$$

FIG. (4) shows the evolution of the CW solution given by Eq. (8). In FIG. 4 (a), we have neglected the FWM nonlinearities and we can see here a formation of a periodic wave train. The period of these pulses is the same as that of the initial perturbation and equal to $2\pi/\Omega$.

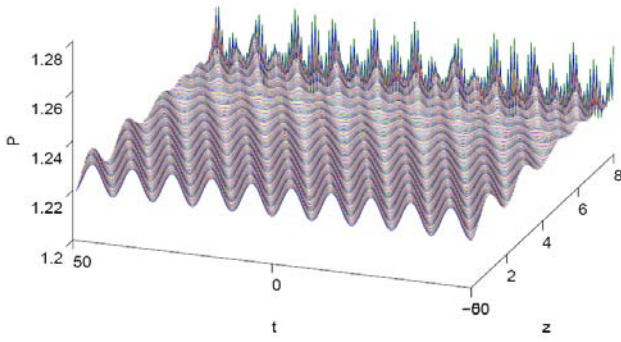


FIG. 4 (a) DIFFERENT PROPAGATION RESULTS WITHOUT THE FWM NONLINEARITIES. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = 0.06\text{ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

A similar result had been obtained in a system of two parallel-coupled fibers modeled by two linearly coupled complex Ginzburg-Landau equations [25] and in a system of higher-order complex Ginzburg-Landau equation, with the fourth-order dispersion and cubic-quintic nonlinear terms [26]. But after a certain distance of propagation, the wave trains become unstable. By comparison, in FIG. 4(b) where the FWM coefficients are taken into account, we observe that the pulse can still propagate stably without any distortion. But at the end of the propagation, the number of the pulses is increased by a factor two.

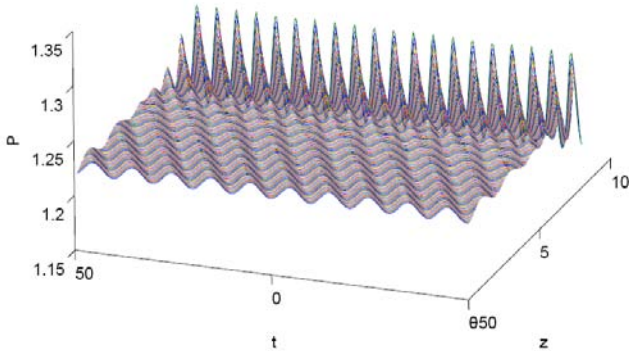


FIG. 4(b) DIFFERENT PROPAGATION RESULTS WITH THE FWM NONLINEARITIES. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = 0.06\text{ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1.

$$\text{Case 2: } \beta_{01}^{+} > 0, \beta_{01}^{-} > 0, \beta_{11}^{+} < 0, \beta_{11}^{-} < 0$$

In FIG. 5 (a) where we have neglected the FWM nonlinearities, we show an example of the first situation where a periodic wave trains is formed. But as in the previous case the pulses are distorted at the end of the propagation.

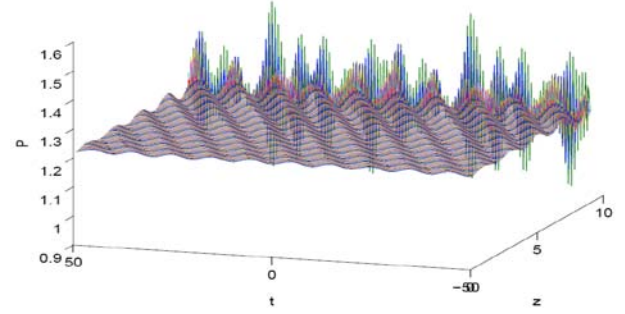


FIG. 5 (a) DIFFERENT PROPAGATION RESULTS WITHOUT THE FWM NONLINEARITIES. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

Another example of the evolution is shown in FIG. 5(b) where the FWM nonlinearities are taken into account. We can see the amplitude of the wave trains increases progressively with the propagation distance. However, contrary to the previous case, the number of the pulses is constant.

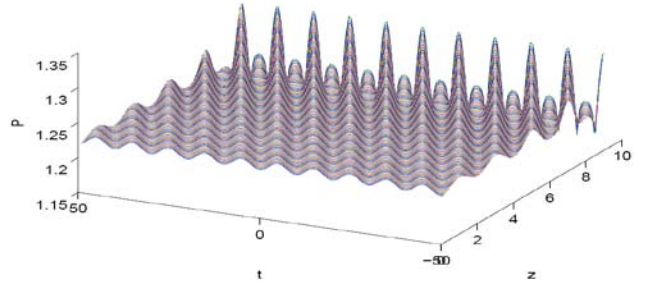


FIG. 5(b) DIFFERENT PROPAGATION RESULTS WITH THE FWM NONLINEARITIES. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = 0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

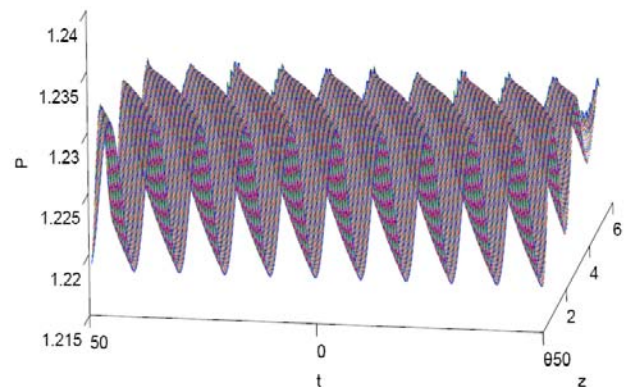


FIG. 6(a) DIFFERENT PROPAGATION RESULTS WITHOUT THE FWM NONLINEARITIES. THE FIBER PARAMETERS ARE $\beta_{01}^{\pm} = -0.06\text{ps}^2/\text{m}$, $\beta_{11}^{\pm} = -0.06\text{ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

Case 3: $\beta_{01}^+ < 0, \beta_{01}^- < 0, \beta_{11}^+ < 0, \beta_{11}^- < 0$

The obtained results are completely different from those obtained in previous case. For example, when we neglect the FWM nonlinearities, we observe in FIG. 6 (a) that at the beginning of the propagation, the waves train are stable. But, at the end of the propagation, these wave trains are distorted.

By taking into account the effect of the FWM nonlinearities in FIG. 6 (b), we observe that the waves train still propagate stably without any distortion.

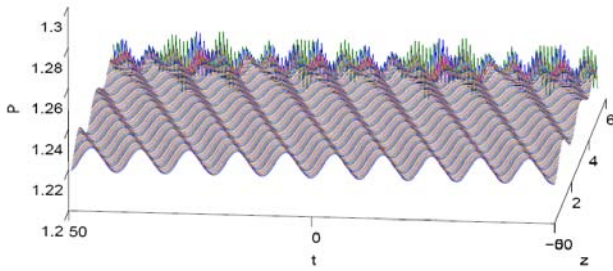


FIG. 6(a) DIFFERENT PROPAGATION RESULTS IN THE PRESENCE OF THE FWM NONLINEARITIES. THE FIBER PARAMETERS ARE $\beta_{01}^+ = -0.06 \text{ ps}^2/\text{m}$, $\beta_{11}^+ = -0.06 \text{ ps}^2/\text{m}$. OTHER PARAMETERS ARE IDENTICAL TO THOSE USED FOR FIG. 1

Conclusions

The full treatment of MI of continuous waves for a system of coherently CNLSE relevant to the polarization of light beams has been studied. In contrast to the previous works, the finite birefringence effect and phase dependent nonlinearity (i.e. the FWM terms) have been considered. The results indicate that both the GVM and the FWM terms will alter the gain spectra of MI considerably in terms of shape, position and especially the number of the spectral regions, but in different ways. Moreover, the nonlinear evolution of MI for the presence of phase dependent nonlinearity has been studied numerically using split-step Fourier method. The most notably change occurs when both beams propagate in the anomalous dispersion regime. We have observed here that when the FWM nonlinearities are omitted, the MI leads to a pattern in the form of periodic pulses array which are distorted at the end of the propagation. However, when the FWM nonlinearities are added, we have obtained that periodic pulses might propagate without any distortion. In order cases in which two beams propagate in the anomalous dispersion regime and the two others in the normal dispersion regime, the situation is quite different. The undistorted pulses are

observed only in the case where the FWM nonlinearities are omitted.

ACKNOWLEDGMENTS

A.M. is grateful for the hospitality of the Max-Planck Institute for the Physics of Complex Systems in Dresden where part of this work was done. K.P. wishes to thank the CSIR, DAE-BRNS, UGC and DST, Government of India, for financial support through major projects.

APPENDIX

Matrix elements of Eq. (7) are given by

$$\begin{aligned}
 m_{11} &= -m_{22} = -m_{44} = m_{33} \equiv -\beta_{01}^{\pm} \Omega^2 + \alpha_{01}^{\pm} \Omega - O_1 P_{01}^{\pm}, m_{12} = m_{34} = m_{23} = -m_{21} = -m_{43} \equiv -O_1 P_{01}^{\pm}, \\
 m_{13} &= m_{31} = m_{32} = m_{14} = -m_{41} = -m_{23} = -m_{24} = -m_{42} \equiv -O_2 \sqrt{P_{01}^{\pm} P_{01}^{\mp}}, \\
 m_{15} &= m_{16} = m_{37} = m_{38} = -m_{25} = -m_{26} = -m_{47} = -m_{48} \equiv -(O_3 \sqrt{P_{01}^{\pm} P_{11}^{\pm}} + O_6 \sqrt{P_{01}^{\pm} P_{11}^{\mp}}), \\
 m_{17} &= m_{18} = m_{35} = m_{36} = -m_{27} = -m_{28} = -m_{45} = -m_{46} \equiv -(O_4 \sqrt{P_{01}^{\pm} P_{11}^{\mp}} + O_5 \sqrt{P_{01}^{\pm} P_{11}^{\pm}}), \\
 m_{51} &= -m_{61} = -m_{62} = m_{73} = m_{52} = m_{74} = -m_{83} = -m_{84} \equiv -(Q_3 \sqrt{P_{11}^{\pm} P_{01}^{\pm}} + Q_6 \sqrt{P_{11}^{\pm} P_{01}^{\mp}}), \\
 m_{53} &= m_{54} = -m_{63} = -m_{64} = m_{71} = m_{72} = -m_{81} = -m_{82} \equiv -(Q_4 \sqrt{P_{11}^{\pm} P_{01}^{\mp}} + Q_5 \sqrt{P_{11}^{\pm} P_{01}^{\pm}}), \\
 m_{55} &= -m_{66} = m_{77} = m_{88} \equiv -\beta_{11}^{\pm} \Omega^2 + \alpha_{11}^{\pm} \Omega - Q_1 P_{11}^{\pm}, m_{56} = -m_{65} = m_{78} = -m_{87} \equiv -Q_1 P_{11}^{\pm}, \\
 m_{57} &= m_{58} = -m_{67} = -m_{68} = m_{75} = m_{76} = -m_{85} = -m_{86} \equiv -Q_2 \sqrt{P_{11}^{\pm} P_{11}^{\mp}},
 \end{aligned}$$

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